

Working hypotheses:

Refer to the figure to the right, where things are represented in zgoubi reference coordinate system:

- the X axis is the electron propagation direction in the Wien filter,
- take $\vec{E} \parallel \vec{Y}$, $\vec{B} \parallel \vec{Z}$, $\vec{v} \parallel \vec{X}$,
- $E_Y(X)$ and $B_Z(X)$ fields are considered step functions in a first part of the simulations, a second part consider effects of fringe fields.

1/ (2 points) Recall the relationship between E_Y and B_Z for a straight electron trajectory.

2/ (2 points) From the analogy between velocity and spin precession equations, namely (with $()' = d()/ds$)

$$\vec{v}' = \vec{v} \times \vec{B}/B\rho \quad \text{and} \quad \vec{S}' = \vec{S} \times \vec{\Omega}/B\rho$$

express the spin precession under the effect of $\vec{E} \times \vec{B}$, in terms of distance s , B_Z , E_Y , v/c , particle rigidity $B\rho$ and Wien filter length L .

3/ (2 points) Take electron energy 350 keV, $L=1.5$ m. Find the numerical values of E_Y and B_Z .

4/ A simulation input file of a 50 cm segment of the spin rotator is provided. This simulation file is in two parts: WFSegment.dat and WFSegment.inc. Running WFSegment.dat (namely: [pathTo]zgoubi -in WFSegment.dat) computes particle and spin motion through the 50 cm Wien filter segment, by stepwise numerical integration (a listing of the execution outcomes is logged in zgoubi.res, always). Read the comments (to the right of the “!” marks) with the Users’ Guide at hand and its INDEX especially, they help understanding the role of the various keywords (OBJET, PARTICUL, WIENFILT, etc.) and their subsequent data lists.

The simulation material includes a gnuplot script: gnuplot_spin.gnu. Read the commands therein, they clarify which data from zgoubi.plt are plotted, and in which columns they are read. See the Users’ Guide, Sec. 8.3, for details regarding the content of the columns in zgoubi.plt.

Obviously, the Wien filter E_Y and B_Z field values in WFSegment.inc are not correct.

4.a (4 points) Confirm this by running the simulation file as is and providing graphs of the electron trajectory and spin precession over the 50 cm segment, reading data from zgoubi.plt. What are the trajectory coordinates and spin rotation angle at the exit of the Wien filter?

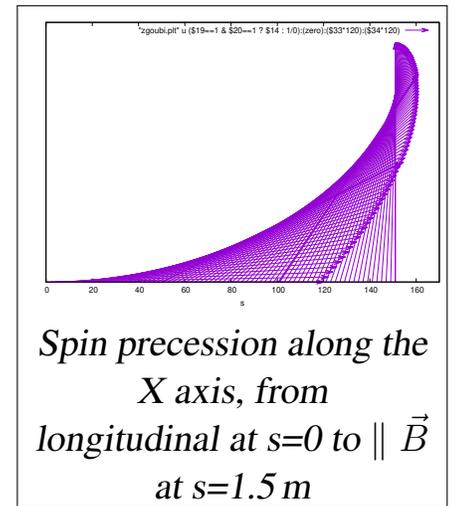
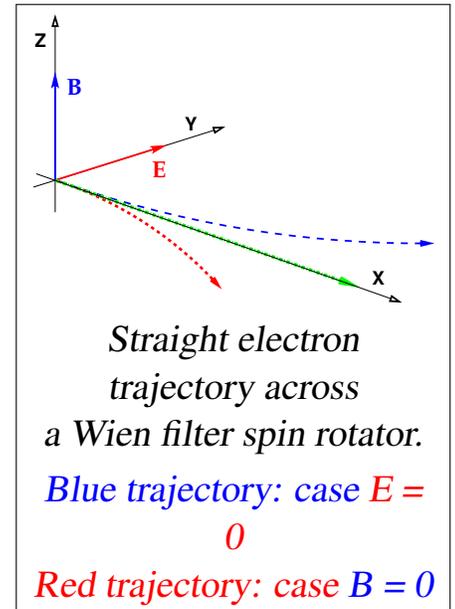
4.b (4 points) Update E_Y and B_Z to their theoretical values.

Provide the new graphs of the electron trajectory and spin motion over the 50 cm segment, so obtained.

4.c (4 points) Using the theoretical E_Y and B_Z values, compute the dependence of the final electron coordinates (position and angle) and spin rotation, on the integration step size. Provide a graph. Explain what you observe.

Hint: use the following form of REBELOTE do-loop command, placed after FIT, to repeat the fitting procedure for a series of integration step size values:

```
'REBELOTE'
100 0.1 0 1          ! Repeat the previous sequence, 100 times, and prior to each repeat,
1                    ! change value of one parameter,
WIENFILT 80 0.01:10. ! namely, number 80 (integration step size) in WIENFILT.
```



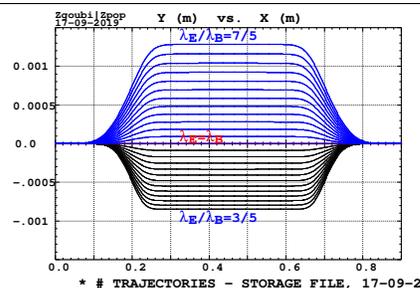
What is the maximum step size for a relative error on spin deviation below 10^{-4} ?

5/ (6 points) Add $\lambda_E = \lambda_S$ length field fall-offs at both ends of the Wien filter segment. This requires extending the numerical integration over an additional distance X_E , upstream of, and X_S , downstream of the Wien filter. Take $X_E = X_S = 20$ cm to ensure cutting off fall-off tails at negligible field values. Ensuring zero particle coordinates at exit and 30 deg spin rotation now requires adjusting the fields.

Uncomment the FIT procedure in the previous file, complete the missing FIT data ('***'), compute the updated values of E_Y and B_Z .

What is there relative difference to the hard edge theoretical values?

Provide graphs of the electron trajectory, and of $E_Y(X)$ and $B_Z(X)$, reading data from zgoubi.plt.



A scan of the on-momentum orbit across the Wien filter, with varying fringe field extent ratio λ_E/λ_B . The orbit is zero at entrance by hypothesis, and zeroed at the exit

6/ (6 points) E and B fringe fields in a Wien filter actually feature different extents. This causes an offset of particle trajectory, see figure to the right.

Keep $\lambda_E = 5$ cm fixed, and vary the magnetic field length in the range $3 \leq \lambda_B \leq 7$ cm, re-adjust the field values to recover null exit coordinates and 30 deg spin rotation angle.

Reproduce the figure on the right: the series of trajectories, $Y(X)$, obtained for this series of values of the ratio λ_E/λ_B .

Provide a graph of the dependence on the ratio λ_E/λ_B , of the relative variation of E_Y and B_Z .

Hint: use the following form of REBELOTE do-loop command, placed after FIT, to repeat the fitting procedure for a series of λ_E/λ_B values:

```
'REBELOTE'
37 0.1 0 1          ! NPASS is of the form int*(7[cm]-3[cm])+1 to allow for lambdaB/lambdaE=1.
2
WIENFILT 22 3.:7.   ! vary lambda_B at entrance EFB from 3 to 7 cm.
WIENFILT 52 3.:7.   ! vary lambda_B at exit EFB from 3 to 7 cm.
```

7/ (10 points)

Give the spinor representation of this Wien filter rotator.

Check consistency with numerical values in the tracking simulations.

(Please use the nomenclature and frame conventions of “Spinor Methods” lecture.)